OSCAR

The OSCAR Team

Bad Münster am Stein, February 18, 2019

Outline

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1. What did we promise?
2. Examples: Current use of the individual cornerstone systems
3. Examples: Current use of more than one cornerstone system

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Long-Term goals

(S1) Integrate all computer algebra systems, libraries and packages developed within the TRR 195 into a visionary next generation open source computer algebra system surpassing the combined mathematical capabilities of the underlying systems. pause

(S2) Boost the performance of the visionary system to a new level by parallelising fundamental algorithms.

Overview: The cornerstones and Julia

Visionary system surpassing the combined capabilities of the underlying systems

GAP: computational discrete algebra, group and representation theory, general purpose high level interpreted programming language.

Singular: polynomial computations, with emphasis on algebraic geometry, commutative algebra, and singularity theory.

Examples:

Multigraded equivariant Cox rings of toric varieties over number fields

Graphs of groups in division algebras

Matrix groups over polynomial rings with coefficients in number fields

Gröbner fans over fields with discrete valuations

polymake: convex polytopes, polyhedral and stacky fans, simplicial complexes and related objects from combinatorics and geometry.

ANTIC: number theoretic software featuring computations in and with number fields and generic finitely presented rings.

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So we talk about interfaced functionality for GAP/Singular/Polymake, and about provided functionality by Nemo/AbstractAlgebra/Hecke

Polymake.jl example

```
julia> p = perlobj( "Polytope",
  Dict( "INEQUALITIES" => [ 0 1 0 ; 0 0 1 ; -1 1 1 ] ) );
julia> l = p.LATTICE_POINTS_GENERATORS;
julia> l[1]
pm::Matrix<pm::Integer>
1 0 1
1 1 0
julia> l[2]
pm::Matrix<pm::Integer>
0 1 0
0 0 1
julia> polytope.intersection( p, polytope.cube( 2 ) ).VERTICES
pm::Matrix<pm::Rational>
1 1 0
1 1 1
1 0 1
```

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- All types of "Big Objects" (e.g., Polytopes, Fans) can be created and manipulated in Julia
- All properties and functions are automatically exported to Julia, and are mirrored as Julia functions

Singular.jl example

```
julia> R, (x, y) = PolynomialRing(QQ, ["x", "y"]);
julia> I = Ideal(R, x + 1, x^2*y + 1)
Singular Ideal over Singular Polynomial Ring
  (QQ),(x,y),(dp(2),C) with generators (x+1, x^2*y+1)
julia> G = std(I)
Singular Ideal over Singular Polynomial Ring
  (QQ),(x,y),(dp(2),C) with generators (y+1, x+1)
julia> fr = fres(G, 0)
..omitted..
julia> fr=minres(fr)
Singular Resolution:
R^1 <- R^2 <- R^1
julia> B=betti(fr)
1×3 Array{Int32,2}:
 1  2  1
```

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- All kernel types and many kernel functions are exported into Julia
- Singular.jl contains abstractions from pure Singular, to make use of Julia's type system and get rid of implicit states (e.g., the current ring)
- Rings defined in Julia can be used as coefficient rings for Singular polynomial rings

Furthermore, Singular.jl will interface the latest Singular features, developed in the TRR

- New non-commutative Groebner basis and algebra in Singular:Plural (Zerz, Levandovskyy, ...)
- Massive shared memory and multi-node parallelization in Singular via pSingular and GPI-Space (Behrends, Böhm, Steenpass, ...)

GAPJulia example

```
julia> S5 = GAP.Globals.SymmetricGroup( 5 )
GAP: SymmetricGroup( [ 1 .. 5 ] )
julia> orb = GAP.Globals.Orbit( S5, 1, GAP.Globals.OnPoints )
GAP: [ 1, 2, 3, 4, 5 ]
julia> g1 = GAP.Globals.GeneratorsOfGroup( S5 )[ 1 ]
GAP: (1,2,3,4,5)
julia> 4^g1
5
```

GAPJulia example, reversed

A GAP lin. comb. of 4620th roots of 1 (about $-3.3 \cdot 10^{-35}$), numerically approximated by arb-library via Nemo: Arb (interval arithmetic):

```
gap> a:= EY(5);;  b:= EY(7);;  c:= EY(11);;  d:= EY(12);;
gap> z:= [ -12230241886849032, -27721673763224765,
>  19808983844326917, 5079707604555803 ] * [a,b,c,d];;
gap> IsPositiveRealPartCyclotomic( z : ShowPrecision );
#I  precision needed: 256
false
```

Creating and factoring a polynomial via Nemo:

```
gap> R := Nemo_PolynomialRing( Nemo_QQ, "x" );;
gap> x := JuliaPointer(Nemo_Polynomial( R, [ 0, 1 ] ));
<Julia: x>
gap> Julia.Nemo.factor(x^10-1);
<Julia: 1 * (x^4-x^3+x^2-x+1) * (x-1) *
       (x^4+x^3+x^2+x+1) * (x+1)>
```

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- Uses a unified GC for GAP and Julia
- All objects can be transparently shared between GAP and Julia
- All functions can be called from either side
- No function call or object handling/conversion overhead

Nemo/Hecke example

```
julia> R, x = PolynomialRing(QQ, "x")
(Univariate Polynomial Ring in x over Rational Field, x)
julia> crt(x-1, x^2+1, x+2, x^2+2)
-3*x^2+x-4
julia> rem(ans, x^2+1), rem(ans, x^2+2)
(x-1, x+2)
julia> S = ResidueRing(R, (x^2+1)*(x^2+2))
Residue ring of Univariate Polynomial Ring in x
   over Rational Field modulo x^4+3*x^2+2
julia> inv(S(x+2))
-1//30*x^3+1//15*x^2-7//30*x+7//15
```

Nemo/Hecke example

```
julia> K, s10 = quadratic_field(10);
julia> c, mc = class_group(K)
(GrpAb: Z/2, ClassGroup map of Set of ideals ...‘
julia> Z_K = maximal_order(K);
julia> P = 2*Z_K + Z_K(s10)*Z_K
<2, s10>
julia> isprime(P), isprincipal(P)
(true, false)
julia> H = number_field(hilbert_class_field(K))
non-simple Relative number field over
  Number field over Rational Field with
    defining polynomial x^2-10
    with defining polynomials ... [x_1^2+(-2)]
```

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- Factorization, (x)gcd, resultant, coprime factorisation, crt, Farey lift
- Ideals

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Advanced features:

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Extension fields

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Benefits from the integration of GAP group theory into Julia!

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All cornerstones are used!

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So using Julia can be used to speed up GAP computations.

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- To achieve that, rigorous interfaces need to be defined, and "mathematical data structures" need to be designed
- So a main goal of OSCAR is providing well-defined, rigorous, and compatible data structures on top of the cornerstone interfaces

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  - Mathematical objects, not only collections of matrices

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So OSCAR will come fully equipped with a graphical interface!

The cube

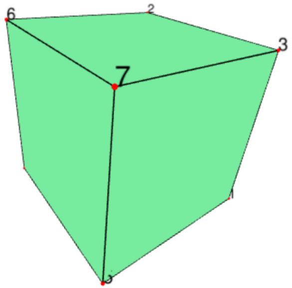
A notebook about Sebastians favorite polymake command

In [3]:
```
c = polytope.cube(3);
```

`c` is a cube in $\mathbb{R}^3$

In [7]:
```
polytope.VISUAL(c)
```

Out[7]:



Future plans

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- Provide releases and training!
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